

## RESONANT OSCILLATIONS OF A GAS IN AN OPEN-ENDED TUBE IN A WEAK TURBULENCE REGIME

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*An analytical theory of resonant oscillations of a gas in an open-ended tube is developed. The gas flow in the tube is assumed to be turbulent. A model of gas flow near the open end of the tube is constructed. This model allows a boundary condition that is free of empirical parameters to be obtained. Theoretical results are in reasonable agreement with experimental data obtained by other authors.*

The theory of resonant oscillations of a gas in tubes-resonators is one of the most interesting problems of hydrodynamic acoustics. High-intensity oscillations of a gas are usually excited by a piston which harmonically oscillates at one end of the tube [1–4]. Of particular interest from the practical viewpoint are open-ended tubes-resonators. Oscillations in such systems are accompanied by a number of interesting effects: an oscillating jet is formed at the open end of a tube [5], a nonuniform temperature field is established in a tube [6], etc.

The qualitative theory of the phenomenon is not yet complete. This is connected with the complexity of the boundary condition at the open end of a tube [7, 8] and poor knowledge of the specifics of an oscillating turbulent flow in a tube [9]. An analytical model of the processes at the open end of the tube has been constructed recently, and, using this model, the boundary condition has been determined [10]. The models of tube turbulence, which were proposed in [11, 12] for the first time, offer a description of experimental results, but have significant drawbacks: (a) they ignore the heat transfer between the tube wall and a gas; (b) they are based on the assumption of a quasi-stationary regime of turbulence; (c) they do not consider dispersion in a turbulent medium.

In this paper, an attempt is made to construct a model of resonant oscillations of a gas at one end of a tube in a turbulent flow regime that is free of the above drawbacks.

Oscillations in a cylindrical tube of length  $L$  and radius  $R$ , which are excited by a harmonically oscillating piston with displacement amplitude  $l_0 \ll L$ , are characterized by the following dimensionless parameters [7–9, 13]:

$$\varepsilon = \frac{V}{\omega L}, \quad H = R\sqrt{\frac{\omega}{\nu}}, \quad \sinh = \frac{\omega R}{V}, \quad M_p = \frac{\omega l_0}{c_0}, \quad Re_\omega = \frac{V^2}{\omega \nu}.$$

Here  $V$  is the amplitude of velocity fluctuations in a velocity loop (for the first resonance, at the open end of the tube),  $\omega$  is the cyclic frequency of oscillations,  $\nu$  is the kinematic viscosity, and  $c_0$  is the speed of sound in an undisturbed gas. Since  $l_0 \ll L$ , for oscillations around the basic resonance frequency  $\omega_0 = \pi c_0 / (2L)$  [13] we obtain  $M_p \ll 1$ . In experiment, we usually have  $H \gg 1$  (the effect of the acoustic boundary layer on the flow core is small) and  $\sinh \leq 1$ . For a long tube ( $L/R \gg 1$ ), the condition  $\sinh \leq 1$  leads to  $\varepsilon \ll 1$ , i.e., the problem can be solved by the methods of disturbance theory [13]. The criterion  $Re_\omega$  indicates a turbulence regime: if  $10^5 \leq Re_\omega \leq 6 \cdot 10^5$ , the regime of weak turbulence occurs [9]. This regime is of interest because the majority of experiments were conducted under these conditions [1–4].

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Nonisentropic oscillations of a gas in a tube can be described by the equations [14]

$$\frac{\partial(\rho u)_s}{\partial t} + \frac{\partial}{\partial x}(\rho u^2)_s + \frac{\partial p}{\partial x} + \frac{2\tau}{R} = 0, \quad \frac{\partial p}{\partial t} + u_s \frac{\partial p}{\partial x} + \bar{\alpha} p \frac{\partial u_s}{\partial x} - \frac{2(\bar{\alpha} - 1)}{R} q = 0, \quad (1)$$

where  $\rho$ ,  $p$ , and  $u$  are the density, pressure, and velocity, respectively;  $\tau$  and  $q$  are the shear stress and the heat flux at the tube wall,  $\bar{\alpha} = c_p/c_V$ ,  $t$  is the time,  $x$  is the longitudinal coordinate (the closed end of the tube corresponds to  $x = 0$ , and the open end to  $x = L$ ), and the subscript  $s$  means that the quantity is averaged over the tube cross section. In the first (acoustic) approximation, we obtain from (1) that

$$\rho_0 \frac{\partial u_{1s}}{\partial t} + \frac{\partial p_1}{\partial x} = -\frac{2\tau_1}{R}, \quad \frac{\partial p_1}{\partial t} + \rho_0 c_0^2 \frac{\partial u_{1s}}{\partial x} = \frac{2(\bar{\alpha} - 1)}{R} q_1. \quad (2)$$

The subscript 1 denotes here the first approximation, and the subscript 0 refers to the quantities of an undisturbed flow. To solve system (2), one must estimate  $\tau_1$  and  $q_1$ .

First of all, we take into account that with allowance for the boundary condition  $u(R) = 0$ , the Reynolds equations written near the tube wall lead to the relation

$$\frac{\partial p}{\partial x} = -\frac{1}{r} \frac{\partial}{\partial r}(r\tau_1) \Big|_{r=R}.$$

It follows from here that, since  $p$  is independent of  $r$ , the dependence of  $\tau_1$  on  $x$  and time is similar to the dependence of the pressure gradient on  $x$ . On the other hand, the Reynolds equations written at the tube axis with allowance for the symmetry condition

$$\frac{1}{r} \frac{\partial}{\partial r}(r\tau_1) \Big|_{r=0} = 0$$

yield

$$\rho_0 \frac{\partial u_1}{\partial t} \Big|_{r=0} + \frac{\partial p_1}{\partial x} = 0,$$

i.e., the dependence of the pressure gradient on  $x$  is similar to that of the velocity. Summarizing the aforesaid, we can show that in (2)

$$\tau_1 \propto u_1(x, t + \phi). \quad (3)$$

The validity of (3) is verified by experiment [9]. For a uniform velocity distribution along the tube, the relationship between the shear-stress amplitude on the wall  $\tau_{1m}$  and the amplitude of velocity fluctuations at the tube axis is as follows [9]:

$$\tau_{1m} = \frac{1}{2} \rho_0 f_w (u_{1m})^2. \quad (4)$$

In the case of weak turbulence, the friction coefficient on the wall is  $f_w \approx 0.005$  [9].

For (3) to be consistent with (4), we assume that

$$\tau_1(x, t) = \rho_0 \beta_0 u_1(x, t + \phi), \quad \beta_0 = \frac{V f_w}{2} \int_0^L \frac{u_{1m}(x)}{V} dx, \quad (5)$$

where the coefficient  $\beta_0$  takes into account that the amplitude of velocity fluctuations  $u_{1m}$  depends on  $x$  for resonant oscillations of a gas in a tube.

Before using (5) in (2), we pass from the flow velocity at the tube axis  $u_1$  to the cross section-averaged velocity  $u_{1s}$  and take into account the phase shift between the oscillations  $\tau_1$  at the wall and  $u_1$  at the axis. In a weak turbulence regime, the amplitude profile of velocity fluctuations is uniform everywhere, except for a layer of thickness  $\delta_1$ , in which a universal distribution of the amplitude of velocity fluctuations is observed, i.e.,

$$\frac{u_{1m}}{u^*} = 2.5 \ln \left( \frac{R-r}{\nu} u^* \right) + 5. \quad (6)$$

Here  $u^* = (\tau_{1m}/\rho_0)^{1/2}$  is the dynamic velocity and  $r$  is the radial coordinate [9]. As follows from experimental data of [9], the layer thickness  $\delta_1$  can be found from the formula

$$\frac{\delta_1}{R} = \frac{0.0154}{\sinh}. \quad (7)$$

Using (6) and (7), we obtain

$$u_{1s} = u_{1m}B, \quad B = 1 - (C \ln Re_\omega + D)/\sinh, \quad (8)$$

where  $C \approx -0.00385$  and  $D \approx 0.0546$ .

The empirical formula for the phase shift  $\phi$  between  $\tau_1$  at the wall and  $u_1$  at the tube axis is obtained from the data of [9]:

$$\phi = 0.838 - 0.891(Re_\omega \cdot 10^{-6}). \quad (9)$$

With allowance for (8) and (9), we have

$$\tau_1 = \rho_0\beta \exp(i\phi)u_{1s}, \quad \beta = \beta_0/B. \quad (10)$$

To estimate  $q_1$ , we assume

$$q_1 = -\beta_T p_1. \quad (11)$$

The thicknesses of the dynamic and thermal boundary layers can be calculated from the formulas

$$\delta_1 = \sqrt{2\mu_e/\rho_0\omega}, \quad \delta_{T1} = \sqrt{2\lambda_e/\rho_0c_p\omega} \quad (12)$$

(the subscript  $e$  refers to the effective value). Since  $\delta_1 \ll R$  and  $\delta_{T1} \ll R$ , in the definitions

$$q_1 = \lambda_e \frac{\partial T_1}{\partial r} \Big|_w, \quad \tau_1 = -\mu_e \frac{\partial u_1}{\partial r} \Big|_w$$

the derivatives with respect to  $r$  can be replaced by the increment ratios. From (10) and (11), and the conditions at the tube wall  $u_1(R) = 0$  and  $T_1(R) = 0$ , it follows that

$$\frac{\lambda_e T_{1m}}{\delta_{T1}} \approx \beta_T p_1, \quad \frac{\mu_e}{\delta_1} \approx \rho_0\beta \exp(i\phi). \quad (13)$$

One can easily show that  $p_1 \approx \rho_0 c_p T_{1m}$  outside the boundary layer. From (12) and (13), we obtain

$$\beta_T = \beta \exp(i\phi)/\sqrt{Pr_t}. \quad (14)$$

The turbulent Prandtl number is  $Pr_t \approx 0.9$  in the boundary layer of steady turbulent flows [15]. We assume this value to be acceptable for our case, too.

We pass to the dimensionless variables in (2), assuming that  $\bar{p}_1 = p_1/\rho_0 c_0^2$  and  $\bar{u}_{1s} = u_{1s}/c_0$ . Taking into account (10), (11), and (14), we obtain

$$\frac{1}{c_0} \frac{\partial \bar{u}_{1s}}{\partial t} + \frac{\partial \bar{p}_1}{\partial x} = -a \bar{u}_{1s}, \quad \frac{1}{c_0} \frac{\partial \bar{p}_1}{\partial t} + \frac{\partial \bar{u}_{1s}}{\partial x} = -\frac{\varkappa - 1}{\sqrt{Pr_t}} a \bar{p}_1, \quad a = \frac{2\beta \exp(i\phi)}{Rc_0}. \quad (15)$$

The solutions of system (15) have the form

$$\bar{p}_1 = r_1 \cos(kx + \alpha_1 + i\gamma_1) \exp[i(\omega t + \psi_1)], \quad (16)$$

$$\bar{u}_{1s} = -ir_1 \mu_1 \sin(kx + \alpha_1 + i\gamma_1) \exp[i(\omega t + \varphi + \psi_1)].$$

Here  $r_1$ ,  $\alpha_1$ ,  $\gamma_1$ , and  $\psi_1$  are real constants of integration,  $\mu_1 = |k/(k_0 - ia)|$ ,  $k_0 = \omega/c_0$ ,  $\varphi = \arg[k/(k_0 - ia)]$ , and

$$k^2 = k_0^2 \left[ 1 - i \frac{a}{k_0} \left( 1 + \frac{\varkappa - 1}{\sqrt{Pr_t}} \right) - \frac{\varkappa - 1}{\sqrt{Pr_t}} \frac{a^2}{k_0^2} \right]. \quad (17)$$

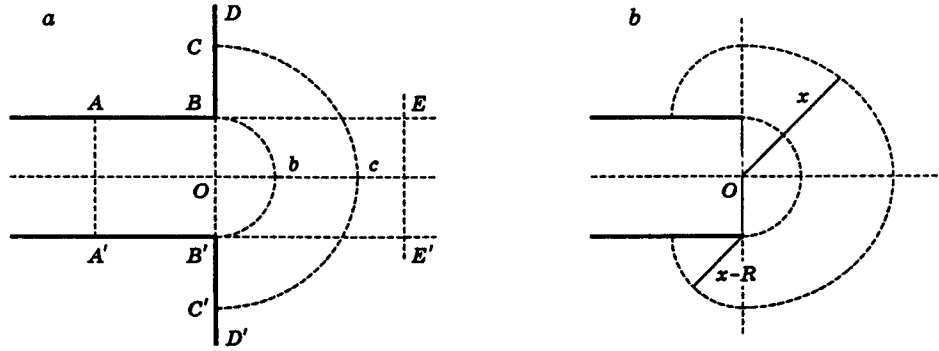


Fig. 1

Since  $a/k_0 \ll 1$ , the last term in (17) can be ignored, i.e.,

$$k \approx k_0 - ib, \quad b = \frac{a}{2} \left( 1 + \frac{\varkappa - 1}{\sqrt{Pr_t}} \right) = b_1 + ib_2, \quad (18)$$

where  $b_1 = (\beta/Rc_0)(1 + (\varkappa - 1)/\sqrt{Pr_t}) \cos \phi$  and  $b_2 = (\beta/Rc_0)(1 + (\varkappa - 1)/\sqrt{Pr_t}) \sin \phi$ . Thus,  $k \approx k_0 + b_2 - ib_1$ ;  $b_1$  and  $b_2$  refer to absorption and dispersion, respectively.

We now consider the boundary conditions. We prescribe the piston velocity at the tube end closed by the piston ( $x = 0$ ):

$$\bar{u}_{1s}(0, t) = -M_p \exp(i\omega t). \quad (19)$$

We consider the flow at the open end ( $x = L$ ) of the tube equipped with an infinite flange (Fig. 1a). Let, at a certain distance from the exit cross section inside the tube (cross section  $AA'$ ), the velocity variation obey the law

$$u = V \cos \omega t. \quad (20)$$

We use a model proposed in [16, 17] which implies the jet outflow ( $u \geq 0$ ) and the spherical inflow ( $u \leq 0$ ) into an orifice located at the point  $O$ . The gas leaving the tube is enclosed in a volume with generatrices  $BE$  and  $BE'$ . Since the mixing layer of the jet has no time to develop near the open end of the tube (at distances  $x < 6R$  [5]), the jet cross-sectional area remains practically constant and equal to the tube cross-sectional area  $S_0$ . In this case, the gas velocity is also independent of  $x$ . The gas flows into the tube through the hemispheres  $BbB'$ ,  $CcC'$ , etc. The role of viscous losses in the source of suction is insignificant [18]. Thus, we can assume that the gas inflow is potential and the hemispheres are isotachs. The amount of gas crossing the hemispheres remains constant, and the following equation is valid for a hemisphere of arbitrary radius  $x$ :

$$u(x, t) = \Phi(x)u_{BbB'}(t) \quad [\Phi(x) = R^2/x^2], \quad (21)$$

where  $u_{BbB'}(t)$  is the velocity at the points of the hemisphere  $BbB'$  passing through the tube edges ( $x = R$ ). For a tube without a flange (Fig. 1b), we have  $\Phi(x) = R^2/[x^2 + (x - R)^2 + (\pi/2)R(x - R)]$ .

We consider the outflow through the cross section  $BB'$  and the inflow through the hemisphere  $BbB'$ . By virtue of the mass conservation law, the amount of gas going out through  $BB'$  should be compensated by a return gas flow through  $BbB'$ , i.e.,

$$S_0 \int_0^{t_1} u_{1s}(t) dt + S \int_{t_1}^T u_{BbB'}(t) dt = 0 \quad (22)$$

( $S = 2\pi R^2$ ). Since  $S > S_0$ , to satisfy (22) it is necessary that the outflow duration  $t_1$  be larger than the inflow duration. This is possible if the velocity has a constant component. Assuming the latter to be proportional to the amplitude of fluctuations of the velocity  $V$ , we obtain [17] the expression  $u = V(m_0 + \cos \omega t)$ ,  $x = R$ ,

where the parameter  $m_0$  should be determined from (22).

The outflow duration  $t_1$  is found from the condition  $u = 0$ . Then we have

$$\begin{aligned} u_{1s}(t) &= BV(m_0 + \cos \omega t), & -(\pi/2 + \theta) \leq \omega t \leq (\pi/2 + \theta), \\ u_{BbB'}(t) &= V(m_0 + \cos \omega t), & (\pi/2 + \theta) \leq \omega t \leq (3\pi/2 - \theta), \end{aligned} \quad (23)$$

where  $\theta = \arcsin m_0$ . Substituting (23) into (22), we obtain

$$(B + 2)\pi m_0 + 2(B - 2)[m_0 \arcsin m_0 + \sin(\arccos m_0)] = 0. \quad (24)$$

The calculation shows that  $B \approx 0.93$  under the experimental conditions of [3]. It follows from (24) that  $m_0 \approx 0.239$ .

We consider the oscillations of the particles intersecting, for instance, the cross section  $EE'$  (Fig. 1a), assuming the motion to be potential. The outflow velocity is determined by (23), and the inflow velocity is found from (21) according to which the velocity decreases rapidly as  $x$  increases. Expanding  $u(t)$  into a Fourier series, we have

$$\begin{aligned} \bar{u} &= M_E \left\{ \left( \frac{m_0}{2 + a_0} \right) + \left( \frac{m_0}{2 - a_0} \right) \Phi(x) + \left[ \left( \frac{1}{2} + a_1 \right) + \left( \frac{1}{2} - a_1 \right) \Phi(x) \right] \cos \omega t + a_2 [1 - \Phi(x)] \cos 2\omega t + \dots \right\}, \\ a_0 &= \frac{1}{\pi} (m_0 \theta + \cos \theta), & a_1 &= \frac{1}{\pi} \left( \theta + 2m_0 \cos \theta - \frac{1}{2} \sin 2\theta \right), \\ a_2 &= \frac{1}{\pi} \left( \cos \theta - m_0 \sin 2\theta - \frac{1}{3} \cos 3\theta \right), & M_E &= \frac{V}{c_0}, & \bar{u} &= \frac{u}{c_0}. \end{aligned} \quad (25)$$

An analysis shows that significant changes in  $\bar{u}$  with distance  $x$ , which are faster in the case of a tube without a flange, end at  $x \approx 3R$ . Beginning from  $x \approx 5R$ , the composition of oscillations becomes independent of  $x$  and the geometry of the tube's open end. For  $x > 6R$ , the viscosity becomes significant. The evolution of an oscillating jet was considered in detail in [5]. Let  $\bar{u}_\infty$  be the velocity at a large distance from the open end of the tube, where we can still ignore viscous effects (let this be valid for the cross section  $EE'$ ). Assuming  $x \rightarrow \infty$  in (25), we have

$$\bar{u}_\infty = M_E [(m_0/2 + a_0) + (1/2 + a_1) \cos \omega t + a_2 \cos 2\omega t + \dots]. \quad (26)$$

For a potential flow whose velocity is described by (25), we can use the Lagrange–Cauchy integral

$$\frac{p}{\rho_0} + \frac{u^2}{2} + \frac{\partial \varphi}{\partial t} = F(t), \quad (27)$$

where the order of the third term is estimated as Sh, i.e., it can be ignored. Applying (27) to two cross sections (for example,  $AA'$  and  $EE'$  in Fig. 1a), assuming the atmospheric pressure in the cross section  $EE'$ , and obtaining the velocity from (20) and (27), after simple transformations we have

$$\bar{p}_1(L, t) = m \bar{u}_{1s}^0 \bar{u}_{1s}(L, t), \quad m = m_1/B^2, \quad m_1 = (1/2 + a_1)(m_0/2 + a_0 + a_2/2), \quad (28)$$

where  $\bar{u}_{1s}^0$  is the amplitude of velocity fluctuations averaged over the tube cross section at the open end of the tube. Under the experimental conditions of [3], we have  $m_1 \approx 0.361$ .

Condition (28) is nonlinear, as in [3, 6–8, 11], but, in contrast to the variants used in these papers, it is *derived* from the flow model near the open end of the tube without any semiempirical considerations.

Substituting the solutions of (16) into (19) and (28), we obtain a system for determination of the desired constants:

$$\begin{aligned} r_1 \mu_1 \sin \alpha_1 \cosh \gamma_1 &= M_p \cos(\varphi + \psi_1), & r_1 \mu_1 \cos \alpha_1 \sinh \gamma_1 &= -M_p \sin(\varphi + \psi_1), \\ \cos z \cosh w &= m r_1 \mu_1^2 \sqrt{\sin^2 z + \sinh^2 w} (\cos z \sinh w \cos \varphi + \sin z \cosh w \sin \varphi), \\ \sin z \sinh w &= m r_1 \eta_1^2 \sqrt{\sin^2 z + \sinh^2 w} (\sin z \cosh w \cos \varphi - \cos z \sinh w \sin \varphi). \end{aligned} \quad (29)$$

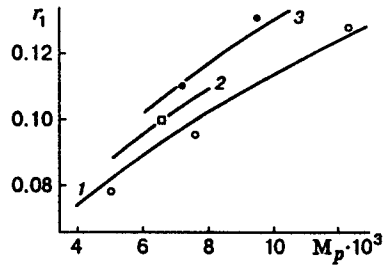


Fig. 2

TABLE 1

| $M_p \cdot 10^3$ | $L/R$ | $\Delta\omega/\omega_0$ |            |
|------------------|-------|-------------------------|------------|
|                  |       | Theory                  | Experiment |
| 5                | 171   | 0.020                   | 0.025      |
| 7.6              | 171   | 0.022                   | 0.033      |
| 12,35            | 171   | 0.023                   | 0.030      |
| 6.6              | 129   | 0.021                   | 0.027      |
| 7.2              | 89    | 0.022                   | 0.023      |
| 9.5              | 89    | 0.022                   | 0.027      |

Here  $z = (k_0 + b_2)L + \alpha_1$  and  $w = \gamma_1 - b_1L$ . System (29) is easily solved under the assumptions  $r_1 \ll 1$ ,  $\sinh w \sim r_1$ ,  $\cosh w \sim 1$ , and  $\mu_1 \sim 1$ , whence  $\varphi_1 \ll 1$ . We write the solution in the form

$$\alpha_1 = \frac{\pi}{2} - (k_0 + b_2)L, \quad \gamma_1 = mr_1 + b_1L, \quad \sin(\varphi + \psi_1) = -\frac{r_1(mr_1 + b_1L) \sin(k_0 + b_2)L}{M_p}, \quad (30)$$

$$r_1 \sqrt{\cos^2(k_0 + b_2)L + (mr_1 + b_1L)^2 \sin^2(k_0 + b_2)L} = M_p.$$

It follows from the above equations that the resonance in the system is reached when  $(k_0 + b_2)L = \pi/2$ , i.e., the resonance frequency is shifted, and, for this shift, from formulas (5), (10), and (18) we obtain

$$\frac{\pi/2 - k_0L}{\pi/2} = \frac{2}{\pi} b_2L = \frac{2}{\pi} \frac{f_w r_1}{3B} \frac{L}{R} \left(1 + \frac{\alpha - 1}{\sqrt{Pr_t}}\right) \sin \phi. \quad (31)$$

Under the resonance conditions, it follows from (30) that

$$r_1 = M_p^{1/2} \left[ m + \frac{f_w}{3B} \frac{L}{R} \left(1 + \frac{\alpha - 1}{\sqrt{Pr_t}}\right) \cos \phi \right]^{-1/2}, \quad (32)$$

which again takes into account (5), (10), and (18).

Figure 2 shows experimental data [3] (points) and calculation results obtained by (32) (curves). Agreement of the data is quite satisfactory: the points deviate from the dependence (32) by no more than 5%. The data scatter is caused, as follows from (32), by different  $L/R$  ratios ( $L/R = 171, 129$ , and  $89$  for curves 1–3, respectively).

We now discuss the shift of the resonance frequency  $\Delta\omega/\omega_0$  described by formula (31). Before comparing the values calculated from (31) with experimental data, we have to take into account the so-called tip correction to the tube length, which is induced by flow inertia near the open end [3], i.e., to substitute  $L$  for  $L + \Delta R$  in (31), where  $\Delta \sim 1$  [3]. For the resonance-frequency shift, we can write

$$\frac{\Delta\omega}{\omega_0} = \frac{2}{\pi} \frac{f_w r_1}{3B} \frac{L}{R} \left(1 + \frac{\alpha - 1}{\sqrt{Pr_t}}\right) \sin \phi + \Delta \frac{R}{L}.$$

The maximum discrepancy between the calculated and experimental results is 33% (see Table 1).

Thus, we can state that the model proposed in the present work is in good agreement with the available experimental data.

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